Application of Lagrangian configuration to the analysis of electrostatically-driven MEMS resonators

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It has been previously shown that Lagrange configuration formalism relying on the so-called material coordinates instead of the spatial ones provides a convenient framework to model the influence of static bias on the propagation of elastic waves [1]. The advantage of this manner of writing the equation of an infinitesimal vibration superimposed on a static or quasi-static bias is that the incremental dynamic propagation equations are associated to a set of boundary conditions where the geometry of the body is defined in a reference state which is the undeformed, stress-free state of the body prior to any bias or vibration. This geometry is perfectly well-defined and known, which may not be the case of the geometry of the body in the so-called initial state [thurston], *i.e.* the biased state, prior to any vibration. The price to pay is the need of writing the linearized equations of motion with help of redefined effective elastic constants, which must be stored in [9x9] matrices since they express the linear relationsship between the asymmetric second Piola-Kirchhoff tensor and the strain gradients.

Some practical applications of this theory were found in the determination of the temperature derivatives of elastic constants [2], and in various problems such as the dynamic thermal sensitivity of resonators submitted to rapidly varying temperature changes, or predicting the transfert function of frequency-output stress sensors. In the first case, the bias is homogeneous, which greatly facilitates the obtention of the effective material constants (elastic, piezoelectric, permittivity). In the second case, the inhomogenous biasing state results into inhomgenous values of the effective material constants. Calling for an appropriate perturbation procedure, for instance based on Green's function [3], allows an easy access to the frequency shift due to the biasing state.

Rather popular in the scientific litterature of the 90's about piezoelectric resonators, this approach was surprisingly ignored in the modelling of MEMS. Nevertheless, it can be very useful to accurately analyze the behavior of electrostatically-driven MEMS resonators, which must be consistently submitted to a biasing state created by a DC polarization voltage to be exploitable in oscillator circuits. This paper proposes an example of application of Lagrange configuration to the modelling of the electrostatically-driven vibrations of a silicon bar in flexure modes. The method is presented with the necessary emphasis on the differences with a more conventional parametric writing of the equations of motion. Because Lagrange writing is appropriate to analyze problems in finite deformation in the framework of non-linear electroelasticity, it is an excellent tool to model electrostatically-driven MEMS for which the non linearity of the voltage-displacement characteristic is at the root of the pseudo-piezoelectric behavior permitting their practical application for the frequency control of electronic oscillators.

[1] J.C. Baumhauer and H.F. Tiersten, *Nonlinear electroelastic equations for small fields superposed on a bias*, J. Acoust. Soc. Am., **54**, 1017-1034, 1973.
[2] R. Bourquin, B. Dulmet, *Thermal sensitivity of elastic coefficients of langasite and langatate*, Trans. on UFFC, **56**, 10, 2079-2085, 2009.
[3] H.F. Tiersten and B.K. Sinha, *A perturbation analysis of the attenuation and dispersion of surface waves*, J. Appl. phys. 49 (1), January 1978.