Least-squares analysis of clock comparison data to deduce optimized frequency and frequency ratio values

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The most advanced optical atomic clocks have now reached levels of stability and accuracy that significantly surpass the performance of the best caesium primary frequency standards, raising the possibility of a future redefinition of the SI second. Optical clocks can already be used as secondary representations of the second, with recommended frequencies and uncertainties being assigned by the CCL-CCTF Frequency Standards Working Group.

Almost all data considered so far by this group comes from absolute frequency measurements of optical clocks relative to caesium fountain primary frequency standards. However future information about the reproducibility of the optical clocks will come mainly from direct optical frequency ratio measurements. For example, within the EMRP-funded ITOC project[1], a co-ordinated clock comparison programme will lead to a set of frequency ratio measurements between all high accuracy optical clocks being developed within European NMIs, as well as a comprehensive set of absolute frequency measurements with uncertainties at the limit set by caesium primary standards. This set of measurements will be over-determined, in the sense that it will be possible to deduce some of the frequency ratios from several different experiments.

Here we describe the development of techniques for analysis of such over-determined sets of clock comparison data. Our approach is based on that used by CODATA to provide a self-consistent set of internationally recommended values of the fundamental physical constants[2]. As input data we take a set of frequency ratio measurements (including optical frequency ratios and optical-microwave frequency ratios on an equal footing), together with their variances and covariances. Each measured frequency ratio is expressed as a function of one or more of a set of adjusted frequency ratios, and a least-squares adjustment performed to determine the best values (and uncertainties) for these adjusted ratios. By accounting for the correlations between the output values, the value and uncertainty of any other frequency ratio can then be calculated. Self-consistency checks on the body of data provide verification of the uncertainty evaluations for each individual clock and enable any issues with individual clocks to be identified.

In this way our analysis software can be used to provide crucial information about the relative performance of different candidates for an optical redefinition of the SI second. It can also be used to deduce optimized values and uncertainties for the absolute frequencies of each optical clock transition relative to the current definition of the SI second, which are required to maximize the potential contribution of optical clocks to international timescales prior to a redefinition.

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[1] H. S. Margolis *et al.*, "International timescales with optical clocks (ITOC)", pp. 908–911 in Proceedings of the Joint European Frequency and Time Forum and International Frequency Control Symposium (EFTF/IFCS), 2013.

[2] P. J. Mohr and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 1998" Rev. Mod. Phys. 72, 351–495 (2000).