Thermal Noise in Sphere Optical Reference Cavities

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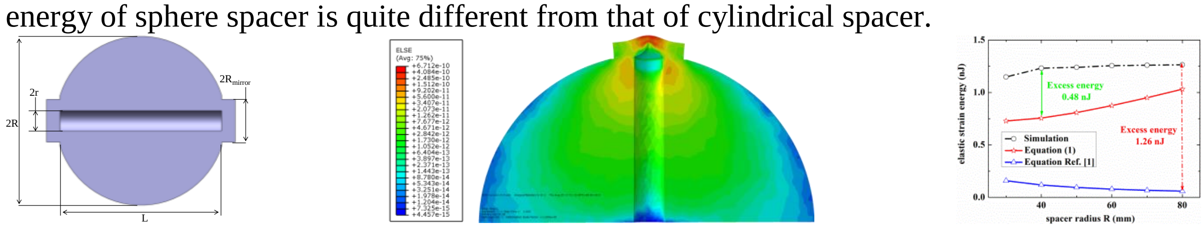
Ultra-stable lasers are essential in many fields. The stability of cavity-stabilized laser is determined by the optical length of the reference cavity. Most environmental perturbations can be suppressed well, including temperature fluctuations and vibrations. A more important issue is that Brownian thermal noise induces optical length fluctuations. To estimate the noise contributions of the spacer, substrates and coatings, Numata derived three equations based on FDT and strain energy. Using these equations, the last two kinds of thermal noise were estimated well.

With the development of space sphere optical cavities, an accurate equation of thermal noise for sphere spacers is essential. However the previous equations in the spacer fit well only for cylindrical spacers. We derive the equation for the sphere spacer based on FDT and strain energy. The equation takes into account axial strain energy and shearing strain energy of the sphere cavity.

The sketch of the sphere cavity is depicted in Fig. 1(a). Since the thermal noise and strain energy in the cavity is the direct proportion relations, we quote our results in terms of strain energy. It is assumed that the shearing stress in the sphere spacer is linear along the direction of the spacer radius. The equation of the strain energy U of the sphere spacer can be derived as follows,

$$U = \frac{F^2}{2\pi E\sqrt{R^2-r^2}}\ln\frac{\sqrt{R^2-r^2}+L}{\sqrt{R^2-r^2}-L} + \frac{F^2(1+u)}{9\pi E}\left(\frac{1}{\sqrt{R^2-r^2}}\ln\frac{\sqrt{R^2-r^2}-L}{\sqrt{R^2-r^2}+L} + \frac{R}{r^2}\ln\frac{R-L}{R+L} - \frac{R^2}{r^2\sqrt{R^2-r^2}}\ln\frac{\sqrt{R^2-r^2}-L}{\sqrt{R^2-r^2}+L}\right) \quad (1)$$

Here u is Poisson ratio, F is the amplitude of an oscillatory force. As the cross section of sphere spacer varies with the cavity length and the shearing strain energy is non-negligible, the strain energy of sphere spacer is quite different from that of cylindrical spacer.



(a) (b) (c)

Fig. 1: (a) Sketch of the sphere cavity. (b) Deformation of ELSE in the pure ULE cavity for a Gaussian pressure profile with a laser beam 240 μm. ELSE denotes the elastic strain energy magnitude for whole element. A beam waist of 2 mm has been applied. (c) Strain energy as a function of spacer radius R . Here R_{mirror} is 12.7 mm.

In Fig. 1 (b), the strain energy for sphere cavity is mostly allocated in the substrate. The majority of the strain energy for spacer is around the central bore near the contact area with the mirrors. A possible reason is the strong stress concentration. The strain energy of ULE spacer as a function of R is shown in Fig. 1 (c). When the equation given in Ref. [1][1] is applied, a huge divergence with the simulation is enhanced when R is larger. This divergence for a spacer radius of 80 mm up to 1.26 nJ can be seen. While the analytic estimation by applying our derived equation (1) is much closer to the simulation for pure ULE sphere cavity. When R is getting larger, the divergence is smaller then. We suppose that the divergence comes from the local deformation of the spacer near the contact surface.

In short, with our derived equation the analytic estimate for the sphere spacer agrees better with simulation. Although it has a minor offset with the simulation, we analyze the possible reasons.

[1] T. Kessler, *et al.*, "Thermal noise in optical cavities revisited", J. Opt. Soc. Am. B **29**(1), 178, 2012.